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Numerical analysis of the influence of the Lode parameter on void growth

K.S. Zhang [a], J.B. Bai [b,*], D. François [b]

[a] *Northwestern Polytechnical University, 710072 Xi'an, People's Republic of China*
[b] *Lab. MSS/MAT, CNRS UMR 8579, Ecole Centrale de Paris, Grande Voie des Vignes 92295 Châtenay Malabry Cedex, France*

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Abstract

In the present paper, three-dimensional numerical analyses of a spherical void, contained within a cubic cell, under finite deformation have been carried out. The calculations took the different stress triaxialities, including the common triaxiality parameter and the Lode parameter, into account. The analyses were focused on the influence of different Lode parameter values on the directional expansion of a void. According to the analyses, it can be concluded that: (1) the void has a different rate of expansion in different directions depending on the variation of the Lode parameter; (2) variation of the Lode parameter changes the critical strain of the void unstable expansion (or void linking) contained within the cubic cell. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Cubic cell; Lode parameter; Triaxiality; Void directional expansion; Critical strain for unstable void expansion

1. Introduction

The mechanical modelling of damage and fracture in ductile materials caused by microvoid evolution has been a field of very active research. Rice and Tracey (1969) suggested an upper limit solution by considering an isolated spherical hole located in an infinite body under remote axisymmetric loading. On this basis, Gurson (1977) considered a spherical void contained within a cell with finite volume and suggested a plastic potential function for porous materials, made of rigid ideal plastic von Mises material and subjected to an arbitrary (not necessary axisymmetric as is often mistakenly thought) loading via conditions of homogeneous boundary strain rate. By adding new parameters to this function, the nucleation, growth and coalescence of the voids can all be taken into account. Tvergaard and Needleman (1995) suggested a modification for the Gurson model to describe plastic flow localisation and the sharp drop of stress in materials after void coalescence. There have been other important extensions, such as the proposition of Leblond et al. (1994), following an earlier suggestion of Pijaudier-Cabot and Bazant (1987). This consisted

* Corresponding author. Fax: +33-1-4113-1430.
E-mail address: baijinbo@mssmat.ecp.fr (J.B. Bai).

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of assuming that porosity (the parameter governing damage and therefore softening) was of a non-local nature, with the use of the spatial convolution integral in its evolution equation.

According to the Rice–Tracey (RT), the Gurson and the Gurson–Tvergaard–Needleman (GTN) models, the growth of a void depends mainly on the stress triaxiality parameter and the effective plastic strain of a cell or an infinite body in which the void is contained. The parameter often used to describe the stress triaxiality is the ratio of the hydrostatic stress over the von Mises stress (Nagaki et al., 1993; Brocks et al., 1995; Koplik and Needleman, 1988; Worswick and Pick, 1990; Kuna and Sun, 1996). When this parameter is not sufficient, the use of the Lode parameter will give a complete description of the stress state. This is especially true when the second principal stress is of major importance.

Lode parameter (which varies from -1 to 1) was first introduced by Walter Lode in 1925 during his study in the university of Göttingen, Germany, who stressed tubes of iron, copper, and nickel, under combined tension and internal pressure (Lode, 1925). The influence of the intermediate principal stress on yielding, and the corresponding failure of Tresca's criterion, was first clearly shown in his work (Hill, 1950 a,b). Relative to this parameter, there are also a Lode parameter defined by the principal strains and a Lode angle which is mainly used in the civil engineering.

It should be pointed out that many results have been obtained by different authors for the axisymmetric stress state where the Lode parameter is always equal to -1. There are also some models using loading via conditions of homogeneous boundary strain rate. The extensions of Gurson model were also made by considering a more general geometry, namely a spheroidal volume containing some spheroidal confocal cavity to incorporate the void shape effect (Gologanu et al., 1996). The influence of the Lode parameter on the evolution of a void in a cell, however, has not been considered explicitly. So, it is necessary to perform the three-dimensional calculation in which the variation of the Lode parameter is taken into account to analyse the evolution of a void contained within a cell (Zhang and Zheng, 1997). This is helpful to have an estimation of the importance of the Lode parameter on the ductile fracture of metals.

2. Cell model

Under the assumption of a periodic microstructure, a porous material can be approximated by cells, each containing a void. One can through such a cell model analyse the constitutive equations and further investigate the complex behaviour of deformation, damage and fracture of a material. The axisymmetric cell model is very convenient because it requires only a two-dimensional calculation, so is the most frequently used model to analyse the expansion of a void.

In this paper, a cubic cell in which a spherical void is contained, as shown in Fig. 1, has been chosen. The initial void volume fraction f_0 under this condition is

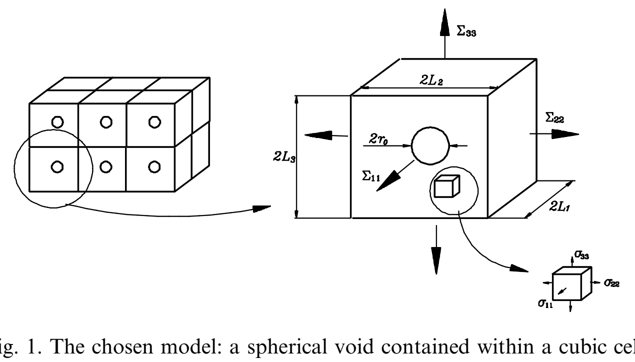

Fig. 1. The chosen model: a spherical void contained within a cubic cell.

$$f_0 = \frac{\pi}{6}\frac{r_0^3}{L_{10}L_{20}L_{30}} \tag{1}$$

The stresses and strains of this cell model are defined by the following equations:

- The principal stresses are given by

$$\left.\begin{aligned}\Sigma_1 &= \tfrac{1}{L_2L_3}\int_0^{L_2}\int_0^{L_3}[\sigma_{11}]_{x_1=L_1}\,\mathrm{d}x_2\,\mathrm{d}x_3\\ \Sigma_2 &= \tfrac{1}{L_3L_1}\int_0^{L_3}\int_0^{L_1}[\sigma_{22}]_{x_2=L_2}\,\mathrm{d}x_3\,\mathrm{d}x_1\\ \Sigma_3 &= \tfrac{1}{L_1L_2}\int_0^{L_1}\int_0^{L_2}[\sigma_{33}]_{x_3=L_3}\,\mathrm{d}x_1\,\mathrm{d}x_2\end{aligned}\right\} \tag{2}$$

- The effective stress and average stress are

$$\left.\begin{aligned}\Sigma_\mathrm{e} &= \tfrac{1}{\sqrt{2}}\sqrt{(\Sigma_1-\Sigma_2)^2+(\Sigma_2-\Sigma_3)^2+(\Sigma_3-\Sigma_1)^2}\\ \Sigma_\mathrm{h} &= \tfrac{1}{3}(\Sigma_1+\Sigma_2+\Sigma_3)\end{aligned}\right\} \tag{3}$$

- The principal strains are

$$E_1 = \ln\left(\frac{L_1}{L_{10}}\right); \qquad E_2 = \ln\left(\frac{L_2}{L_{20}}\right); \qquad E_3 = \ln\left(\frac{L_3}{L_{30}}\right) \tag{4}$$

- The effective strain is

$$E_\mathrm{e} = \frac{\sqrt{2}}{3}\int_0^t\left[\left(\dot{E}_1-\dot{E}_2\right)^2+\left(\dot{E}_2-\dot{E}_3\right)^2+\left(\dot{E}_3-\dot{E}_1\right)^2\right]^{1/2}\mathrm{d}t \tag{5}$$

- The Lode parameter is

$$\mu_\sigma = \frac{2\Sigma_2-\Sigma_1-\Sigma_3}{\Sigma_1-\Sigma_3} \tag{6}$$

- The stress triaxiality parameter is

$$T = \frac{\Sigma_\mathrm{h}}{\Sigma_\mathrm{e}} \tag{7}$$

In general, the triaxial stress state at an arbitrary point of a material can be described by three values. This may be the three principal stresses or the three stress invariants or any three quantities arbitrarily derived from the invariants, for instance the average stress Σ_h, the effective stress Σ_e and the Lode parameter μ_σ. Instead of Σ_h and Σ_e, the ratio T is commonly introduced as a measure of the triaxiality of the stress state. For the axisymmetric stress state, the Lode parameter $\mu_\sigma = -1$, for all values of the stress triaxiality parameter T. Other than this case the Lode parameter μ_σ can have any value between -1 and 1. Those authors only dealing with axisymmetric conditions have thus ignored any possible influence of the Lode parameter. So, it is necessary to investigate the influence of the variation of the Lode parameter μ_σ on the growth of the void, for a general state of stress.

As a preliminary investigation, this paper only discusses three cases:

$\mu_\sigma = 1$ and the middle principal stress is equal to the largest principal stress $\Sigma_1 = \Sigma_2 > \Sigma_3$, and $T = 1$, $1\frac{2}{3}$, 3;

$\mu_\sigma = 0$ and the middle principal stress is equal to the average stress $\Sigma_2 = \Sigma_\mathrm{h}$, for the case of plane strain state, and $T = 1$, $1\frac{2}{3}$, 3;

$\mu_\sigma = -1$ and the middle principal stress is equal to the third principal stress $\Sigma_2 = \Sigma_3 < \Sigma_1$, for the case of an axisymmetric stress state, and $T = 1,\ 1\frac{2}{3},\ 3$.

In general, $T = 1$ corresponds to the stress state at the centre of the minimum cross-section of a round bar undergoing severe necking deformation, $T = 1\frac{2}{3}$ corresponds to the stress state at the centre of the minimum cross-section of a round notched bar at the beginning of tensile deformation, and $T = 3$ corresponds to the stress state at the tip of a mode-I-type crack.

3. Mesh, boundary conditions and material

Considering symmetry, the finite element mesh for a cubic cell is designed as shown in Fig. 2. It consists of a total of 162 20-node isoparametric hexagonal elements and 922 nodes.

The boundary conditions can be described as:

$$\left.\begin{array}{llllll} u_{i1} = 0, & x_1 = 0; & u_{i2} = 0, & x_2 = 0; & u_{i3} = 0, & x_3 = 0 \\ u_{i1} = U, & x_1 = L_1; & u_{i2} = V, & x_2 = L_2; & u_{i3} = W, & x_3 = L_3 \end{array}\right\} \tag{8}$$

where u_{i1}, u_{i2} and u_{i3} are the displacements of node i in the direction x_1, x_2 and x_3 respectively; W is the value chosen in the calculation, it is the elongation displacement along the x_3 axis; U and V are respectively the displacements along the x_1 and x_2 axes, which are calculated according to the given parameters T and μ_σ. In this investigation, the initial edge lengths of the cubic cell are assumed as $L_{10} = L_{20} = L_{30}$.

The program ADINA was used to perform the 3D numerical calculation. The stress-state parameters T and μ_σ were controlled by adding a user's subroutine. The error of the parameters T and μ_σ was limited to

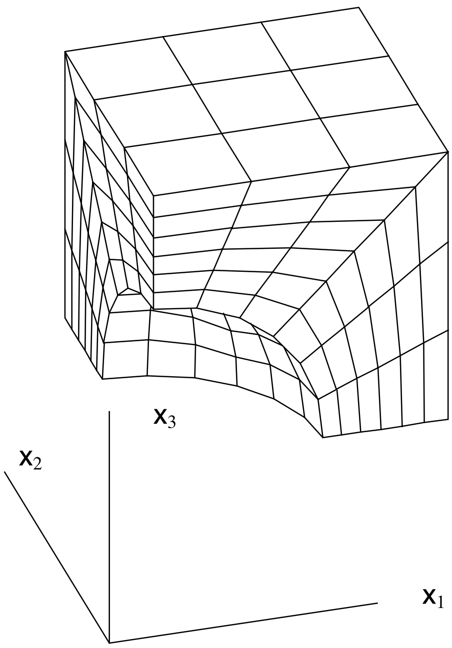

Fig. 2. The finite element mesh of a spherical void contained within a cubic cell.

Table 1
The mechanical properties of the cell matrix

E	v	σ_s	C	n
200 GPa	0.3	500 MPa	910 MPa	0.1

less than 1%. The procedure is to choose first the W value, then calculate U and V according to the T and μ_σ parameters.

The mechanical properties of the cell matrix material are shown in Table 1. E is Young's modulus, v is Poisson's ratio, σ_s is the yield stress, C is the strain hardening coefficient, and n is the strain hardening. They are used to determine the strain hardening rate parameter needed in Prandtl–Reuss equation.

The initial void volume fraction f_0 was taken as 10%.

4. Results

4.1. Influence of the Lode parameter on the deformation of the void and of a cubic cell containing a void

According to the three-dimensional calculations of the cubic cell, the type of deformation of the cell is very different for Lode parameters μ_σ varying from -1, 0 to 1. The difference caused by differences in the values of μ_σ depends significantly on the stress triaxiality parameter T. The larger the value of T, the smaller the influence of μ_σ. For $T = 1$, the influence of μ_σ is very large. Fig. 3 shows the deformation pattern of the cell for $T = 1$ and $\mu_\sigma = -1$, 0 and 1 respectively, at the beginning of the unstable expansion. From this figure, it can be seen that the shape of the deformed void and cell are very different, under the same stress triaxiality parameter T. For the cases of $T = 1\frac{2}{3}$ and $T = 3$, the difference in shape of the two patterns of deformation is still noticeable but significantly smaller.

In order to illustrate the influence of μ_σ on the void shape change, a parameter $a_i(\mu_\sigma)$ is defined as follows:

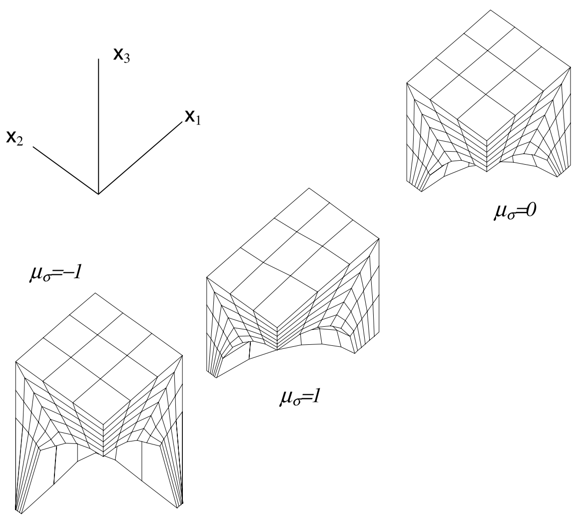

Fig. 3. The deformation type of the void contained within a cubic cell for $T = 1$ with different values of the Lode.

$$a_i(\mu_\sigma) = \frac{(a_i - a_{i0})}{a_{i0}}\Bigg| \tag{9}$$

where subscript i stands for the axes x_1, x_2 and x_3, and where a_i and a_{i0} are respectively the current and initial polar radii in the i direction. Fig. 4 shows the conditions that the evolution of parameter $a_i(\mu_\sigma)$ is very

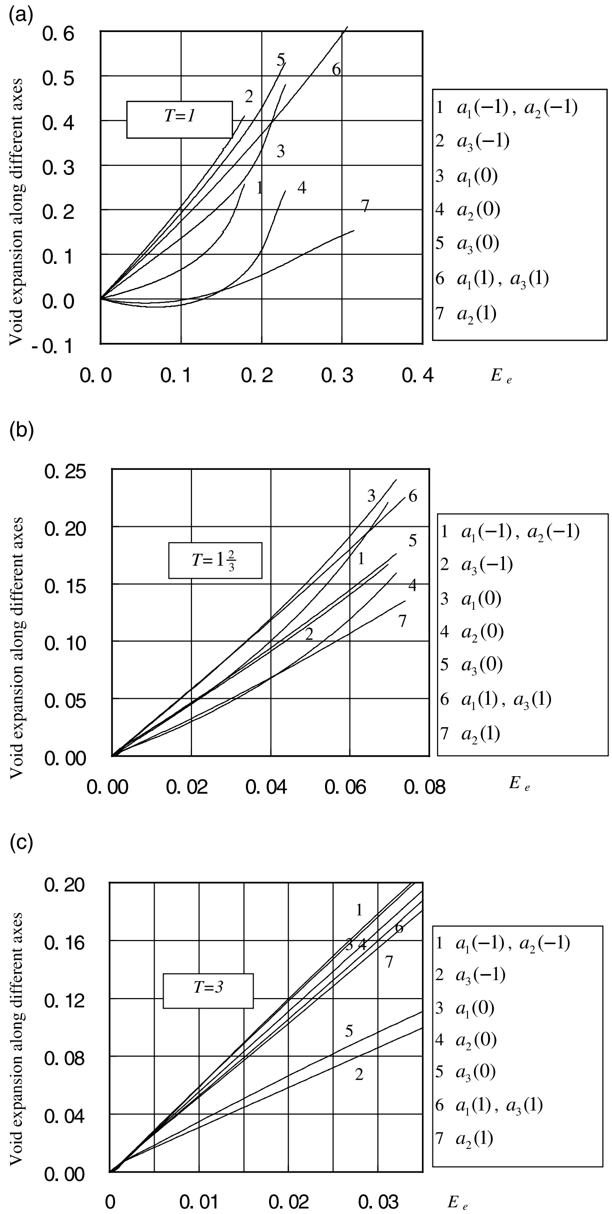

Fig. 4. (a) Void expansion along different axes for different values of the Lode parameter for $T = 1$. (b) Void expansion along different axes for different values of the Lode parameter for $T = 1\frac{2}{3}$. (c) Void expansion along different axes for different values of the Lode parameter for $T = 3$.

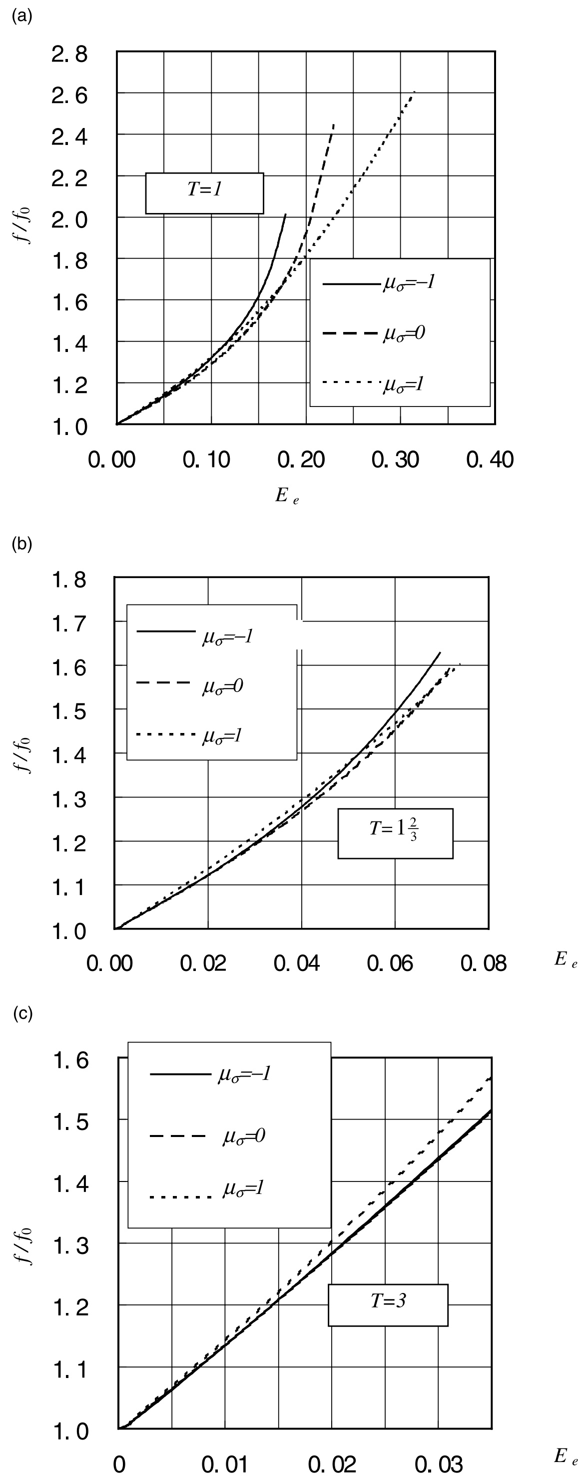

Fig. 5. (a) The evolution of void volume fraction with different Lode parameters for $T = 1$. (b) The evolution of void volume fraction with different Lode parameters for $T = 1\frac{2}{3}$. (c) The evolution of void volume fraction with different Lode parameters for $T = 3$.

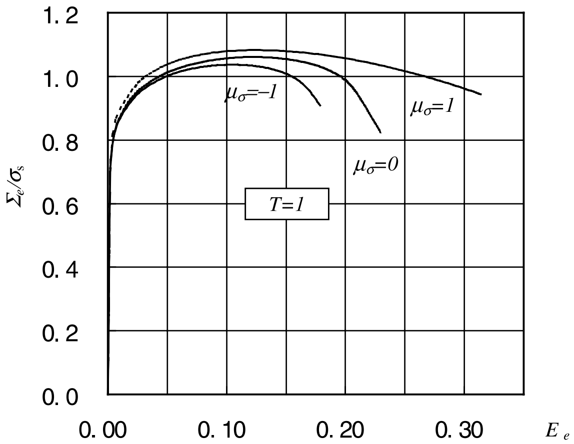

Fig. 6. The influence of the Lode parameter μ_σ on the loss in stress carrying capacity for $T = 1$.

different relative to different μ_σ under the same stress triaxiality parameter T. Let $\Sigma_2 = \Sigma_{11}$, $\Sigma_3 = \Sigma_{22}$, and $\Sigma_1 = \Sigma_{33}$. For $\mu_\sigma = -1$, $\Sigma_2 = \Sigma_3 < \Sigma_1$ and $U = V$, this leads to $a_1(-1) = a_2(-1)$, and they evolve very differently in comparison with $a_3(-1)$. For $\mu_\sigma = 0$, we have $\Sigma_3 < \Sigma_2 < \Sigma_1$, and $\Sigma_2 = \Sigma_\mathrm{h}$, then $a_1(0), a_2(0)$ and $a_3(0)$ are different from each other. For $\mu_\sigma = 1$, it means $\Sigma_3 < \Sigma_2 = \Sigma_1$ and $U = W$, this leads to $a_1(1) = a_3(1)$, and they have a very different evolution law from $a_2(1)$. So, it can be concluded that the expansion of the void can be very different in different directions under the same triaxiality parameter T. From Fig. 4(a)–(c) it can also be seen that the stress triaxiality parameter T can significantly modify the void deformation shape. Quantitatively, these trends are in agreement with those of Cologanu et al. when they incorporated void shape effect into Gurson model (Gologanu et al., 1996).

Fig. 5 shows the evolution of void volume fraction when the loading applied to the cell is controlled in different stress triaxialities, for when T is 1, $1\frac{2}{3}$ or 3, and when the Lode parameter μ_σ is taken as -1, 0 or 1. From these figures, it appears that there is no significant difference in the change of the void volume fraction until the void expands into the unstable stage. But according to Fig. 5(a), it can be seen that variation of the Lode parameter causes a change in the critical strain for void instability. This result can also be observed in Fig. 6. When the Lode parameter has the minimum value, that is when $\mu_\sigma = -1$, the stress carrying capacity of the cell will be lost very quickly. Whereas when the parameter has the maximum value, that is when $\mu_\sigma = 1$, the stress carrying capacity of the cell is lost much later.

4.2. Influence of Lode parameter on void coalescence

For a metallic material with a ductile matrix and voids, its failure is considered to be caused by unstable void expansion which leads to the voids coalescing. During the period when the voids are approaching each other, the necking process is accelerating in the ligaments between the voids. Then void growth in the transverse direction becomes unstable. Thus the void volume fraction in the material will increase rapidly even if the effective strain increases very slowly. In this condition the rapid drop in the stress carrying capacity of the material can be found from the effective stress–strain curve of the cell, as shown in Fig. 6. The value of the Lode parameter has a very significant influence on the rapid loss in the stress carrying capacity of the material. This means that void coalescence depends not only on the stress triaxiality parameter and plastic strain, but also on the value of the Lode parameter. Under certain conditions, keeping the stress triaxiality constant, the different Lode parameter values can cause differences in the value of the fracture strain of one or more times. Fig. 6 shows the cases where the Lode parameter has the values of –1, 0 and 1, respectively.

5. Discussion and conclusions

In the present paper, three-dimensional analyses of a spherical void contained within a cubic cell under different stress states have been carried out. As a preliminary investigation, the conditions where the stress triaxiality parameter $T = 1$, $1\frac{2}{3}$ and 3, and the Lode parameter $\mu_\sigma = -1$, 0 and 1 were considered. The analyses were mainly focused on the influence of different values of the Lode parameter on the directional expansion of a void within a cubic cell.

It is necessary to point out that the influence of the Lode parameter on the evolution of ductile damage in metallic materials has rarely been investigated. Most papers consider the stress triaxiality parameter T as the only stress-state variable to describe the stress circumstance of a void, and the RT model, the Gurson model and the GTN model neglect the influence of the Lode parameter. According to the investigation described in this paper, this influence on the deformation pattern of the void and the cell is very large. The directional expansion of the void can strongly influence the void shape change and the stress and strain distribution in the ligament between voids. Therefore knowledge of the manner of void evolution for different values of Lode parameter and triaxiality parameter, obtained by using three-dimensional analysis, is important in order to understand the damage and rupture mechanisms of ductile porous materials.

According to the results of the calculations made in this paper, the following conclusions can be obtained:

1. Under conditions in which the stress triaxiality parameter T is kept constant differences in the Lode parameter cause a significant difference in void and cell deformation pattern. The damage in the material depends directionally on the Lode parameter.
2. Differences in values of the Lode parameter lead to different coalescence strains. Under certain conditions, these differences can cause the coalescence strain to change by a factor of two.
3. The influence of the Lode parameter is much stronger on void shape than on void volume fraction.
4. It would be necessary to introduce the Lode parameter factor in any new model of ductile fracture.

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